Young Children’s Metarepresentational Competence in Data Modelling

Lyn English

Queensland University of Technology
<l.english@qut.edu.au>

This paper reports findings from an activity implemented in the final year of a 3-year longitudinal study of data modelling across grades 1-3. The activity engaged children in designing, implementing, and analysing a survey about their new playground. Data modelling involves investigations of meaningful phenomena, deciding what is worthy of attention (identifying complex attributes), and then progressing to organising, structuring, visualising, and representing data. The core components of data modelling addressed here are children’s structuring and representing of data, with a focus on their display of metarepresentational competence (diSessa, 2004). Such competence includes students’ abilities to invent or design a variety of new representations, explain their creations, understand the role they play, and critique and compare the adequacy of representations. Reported here are the ways in which the children structured and represented their data, the metarepresentational competence displayed, and links between their metarepresentational competence and conceptual competence.

Introduction

Young children are immersed in our data-driven society, with early access to computer technology and daily exposure to the mass media. The need to advance children’s statistical reasoning abilities, from the earliest years of schooling, has thus been stressed in recent years (e.g., Langrall, Mooney, Nisbet, & Jones, 2008; Shaughnessy, 2010; Whitin & Whitin, 2011). One approach to enhancing children’s statistical abilities is through data modelling (English, 2010; Lehrer & Romberg, 1996; Lehrer & Schauble, 2007).

Data modelling is a developmental process, beginning with young children’s inquiries and investigations of meaningful phenomena, progressing to identifying various attributes of the phenomena, and then moving towards organising, structuring, visualising, and representing data (Lehrer & Lesh, 2003). Data modelling should be a fundamental component of early childhood curricula, yet there exists limited research on such modelling and how it can be fostered in the early school years. The majority of the research has been concerned with the secondary and tertiary levels, with the assumption that primary school children are unable to develop their own models and sense-making systems for dealing with complex situations (Greer, Verschaffel, & Mukhopadhyay, 2007).

Recent research, however, has indicated that young children do possess many conceptual resources which, with appropriately designed and implemented learning experiences, can be bootstrapped toward sophisticated forms of reasoning not typically seen in the early grades (e.g., Clements, Sarama, Spitler, Lange, & Wolfe, 2011; English & Watters, 2005; Papic, Mulligan, & Mitchelmore, 2011; Perry & Dockett, 2008). Most research on early mathematics and science learning has been restricted to an analysis of children’s actual developmental level, which has failed to illuminate their potential for learning under stimulating conditions that challenge their thinking (Ginsburg, Cannon, Eisenband, & Pappas, 2006; Perry & Docket, 2008). Data modelling provides rich opportunities to advance young children’s statistical development and reveal their capabilities in dealing with challenging tasks.

In J. Dindyal, L. P. Cheng & S. F. Ng (Eds.), Mathematics education: Expanding horizons (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia). Singapore: MERGA.

I now address some core components of data modelling, with a focus on structuring and representing data together with the role of metarepresentational competence in children's creation of data models.

Structuring and Representing Data

Models are typically conveyed as systems of representation, where structuring and displaying data are fundamental; the structure is constructed, not inherent (Lehrer & Schauble, 2007). However, as Lehrer and Schauble indicated, children often have difficulties in imposing structure consistently and often overlook important information that needs to be included in their representations or alternatively, they include redundant information. Providing opportunities for young children to structure and display data in ways that they choose, and to analyse and assess their representations is important in addressing these early difficulties. Yet young children's typical exposure to data structure and displays has been through conventional instruction on standard forms of data representation.

As children construct and display data models, they generate their own forms of inscription. By the first grade, children already have developed a wide repertoire of inscriptions, including common drawings, letters, numerical symbols, and other referents. As they invent and use their own inscriptions, children also develop an "emerging meta-knowledge about inscriptions" (Lehrer & Lesh, 2003). Children's developing inscriptional capacities provide a basis for their mathematical activity. Indeed, inscriptions are mediators of mathematical learning and reasoning; they not only communicate children's mathematical thinking but they also shape it (Lehrer & Lesh, 2003; Olson, 1994). As Lehrer and Schauble (2006) stressed, developing a repertoire of inscriptions, appreciating their qualities and use, revising and manipulating invented inscriptions and representations, and using these to explain or persuade others, are essential for data modelling.

Metarepresentational Competence

Children's use of inscriptions plays an important role in their development of metarepresentational competence (diSessa, 2004; diSessa, Hammer, Sherin, & Kolpakowski, 1991). Such competence includes students' abilities to invent or design a variety of new representations, explain their creations, understand the role they play, and critique and compare the adequacy of representations. The use of "meta" to describe these capabilities, as diSessa's emphasises, is to indicate that no specific representational skills are implied. Unlike the standard representational techniques students might have learned from specific instruction, metarepresentational competence encompasses students' "native capacities" (diSessa, 2004, p. 294) to create and re-create their own forms of representation. Their skills here are more "broadly applicable, more flexible and fluid" (diSessa et al., 1991, p. 118), and are not just confined to a narrow set of instructed representations. In essence, diSessa and Sherin (2000) and colleagues coined the term *metarepresentational* to "describe the full range of capabilities that students (and others) have concerning the construction and use of external representations "(p. 386). Indeed, diSessa's research has shown that students do possess a "deep, rich, and generative" understanding of representations, which seems to exist before instruction and is independent of it (p. 387). It appears students are particularly strong at inventing and modifying representations, which diSessa (2004) refers to as "hyper-richness."

Another issue that has received limited attention with respect to children's metarepresentational competence is the joint development of metarepresentational and conceptual competence (diSessa, 2004). As diSessa noted, research is limited here and the role of student-created representations in conceptual development is rather complex. Questions needing attention include how certain strengths or limits of metarepresentational competence might advance or hinder conceptual development, and whether metarepresentational competence and conceptual development develop jointly. Specifically, for the third-year activity reported here, I consider how the children structured and represented their data, and how they displayed metarepresentational and conceptual competence.

Methodology

The participants were from an inner-city Australian school. In the first year of the study, three classes of first-grade children (2009, mean age of 6 years 8 months) and their teachers participated. The classes continued into the second year of the study and finally, two classes continued into the third year (mean age of 8 years 8 months, n=39). A seventh-grade class (age range of 12-13 years) also participated in one of the activities during the second year and shared their models with one of the grade two classes. For all activities, the children worked in small groups.

Design

A teaching experiment involving multilevel collaboration (at the level of student, teacher, and researcher) was adopted throughout the study, where the developing knowledge of all participants was the focus (Lesh & Kelly, 2000). Such an approach is concerned with the design and implementation of experiences that maximise learning at each level. The teachers' involvement in the research was vital; hence regular professional development meetings were conducted. This paper addresses aspects of the student level of development.

Procedures

Literature, both purposefully created and commercially available, was used as a basis for the problem context in each of the activities implemented across the three years. It is well documented that storytelling provides an effective context for mathematical learning, with children being more motivated to engage in mathematical activities and displaying gains in achievement (van den Heuvel-Panhuizen & van den Boogaard, 2008). For the current activity, the storybook, *Hot Cha Cha* (Nobisso, 1998) was initially read to the children. The activity, *Investigating and Planning Playgrounds*, was then introduced, with the children firstly posing questions that might help them find out more about their classmates' thoughts on their new playground. In their groups, the children then created four survey questions and were to provide four answer options for each question (e.g., one group posed the question, *How long do you spend on each piece of equipment*? with the response options of *30 mins, 15 mins, 5 mins*, and *20 mins*). On answering their own questions, each group chose one focus question to which the other groups were to respond. The children were to initially predict how their focus question might be answered by the remaining groups. Each group subsequently analysed all their collected data for their focus question and were to display their findings using their choice of representation. The children were encouraged to represent their findings in more than one way, with no specific direction given. They were supplied with a range of recording material including blank chart paper, 2.5cm squared grid paper, and chart paper displaying a circle shape. The children could use whatever of these materials they liked; no encouragement was given to use any specific recording material. On completion of the activity, the groups reported back to their class peers on their final data models.

Data Collection and Analysis

In each of the third-grade classrooms, two focus groups (of mixed achievement levels and chosen by the teachers), were videotaped and audiotaped. Altogether there were nine groups of children, five in one class and four in the other, who completed the activity. All artifacts were collected and analysed, along with the transcripts from the video and audio tapes. Iterative refinement cycles for analysis of children's learning (Lesh & Lehrer, 2000) were used, together with constant comparative strategies (Strauss & Corbin, 1990) in which data were coded and examined for patterns and trends.

The analysis of the children's transcripts and artifacts took into consideration: (a) how the children structured and represented their data; (b) the "completeness (shows all relevant information)," and "compactness (better use of space)" diSessa, 2004, p. 313) of their representations; and (c) evidence of conceptual and metarepresentational competence.

Selection of Findings

Prior to addressing some findings, it is worth noting that the teachers had done minimal instruction on statistical representations and had not introduced the children to more difficult representations such as circle graphs or line graphs.

How Children Structured and Represented their Data

Of the nine student groups, seven created two or more representations, with one group creating four representations. Vertical bar graphs using the 2.5 cm squared grid paper and circle graphs were the most popular, with seven groups creating a vertical bar graph and seven, a circle graph, with six groups creating both representations. Beginning with a blank sheet of paper, two groups created a vertical bar graph and one group, a line graph. One group made use of a list of their response options and used tally marks to collate their data prior to representing it. Two groups chose unusual representations, such as the "heart monitor" representation created by James' group. As James explained, "Okay, how are we going to show it our own way?! Let's think, just think. I know, like at the hospital, so like... up here (indicates a line graph similar to a heart rate monitor display). So first we're going to may be do about 3cm" (he places his ruler horizontally across the middle of the paper). Another group member further added: "We're doing like a doctor's sort of thing how they go like that (indicating a rise and fall)...like a doctor does...yes, how they have those lines, so we're sort of doing it with maths though."

Completeness and Compactness of the Representations

To determine the completeness of the representations, consideration was given to children's use of inscriptions, and for compactness with respect to the bar graphs, their use of space. Of the 10 bar graphs created (one group completed two), seven displayed inscriptions on both the vertical and horizontal axes, while the remainder omitted numbering the vertical axis, relying instead on a visual counting of the squares on their grid paper. Six bar graphs featured spacing between the drawn bars, presumably to facilitate reading of the graph, with one group commenting, "I'll leave a space so it is neat." Interestingly, two groups who used the grid paper in a landscape position realised they did not have sufficient squares to represent the data for a couple of their response options and hence chose to colour two columns for the one response. All bar and line graphs featured colour, with one group debating whether, in creating a new representation, the same colours should be used ("We colour it exactly the same, and then we're going to colour code exactly as we did on that" [bar graph]; another group member disagreed, however, stating, "We don't need to.")

For those groups that created circle graphs, all used inscriptions to indicate the proportion of each response option. These inscriptions included labelling of the option or using a colour key (two instances of the latter), as well as recording the number of each response and/or recording a percentage (five instances of the latter). The children's use of percent and percentage was an interesting finding, as explained next.

Conceptual and Metarepresentational Competence

There was evidence of development of conceptual and metarepresentational competence as the children created their representations, especially with their circle graphs as they had not been taught how to construct these. The use of a ruler and/or estimation was evident in each of the focus groups, as they tried to represent their data. For example, one group argued over how to estimate a sector for each response option, with one child insisting that "You have to find the middle first. That's the first thing you actually do." He then placed his ruler through the centre of the circle and drew a small sector to represent the two "for exercise" responses to the focus question, "Why do you like the equipment you chose?" He explained, "Two will only be like this (drawing a small sector)...cause it's a very small amount." He then recorded "2" in the sector he had created. When asked how many "pieces of the pie" they needed, the group quickly replied "four, cause there's four of them (response options)." They also commented that the four sectors would not be the same size "because if there's two people, this would have to be a smaller piece to fit two people and a bigger piece to fit nine people in." In estimating the size of the sector to represent the nine responses of "it's challenging," one child claimed that "nine would be half of it" (there was a total of 20 responses to their focus question). After much discussion, the group decided "no, no, no, that nine can't be that big (half of the circle)" taking into account the frequencies of the other response options (six, three, and two). One child subsequently tried to measure the sectors with his fingers to make the nine sector smaller than the total of the other response options (11), explaining, "Yeah, it actually does have to be a bit smaller."

Another display of the children's conceptual and metarepresentational competence occurred when they commented that their second representation would display the same data. After completing a bar graph, one student group decided to construct a circle graph to display their findings from their focus question, "Why do you like the playground?":

> Peter: You just do the same. It's a pie.
>
> Belinda: So how many people are there…
>
> Kim: But we need to do the biggest one (sector) for the most amount of votes.
>
> Peter: We need to write, we do 12 for the most amount of votes; we take the data that we've got from here (bar graph) and we write what it is.
>
> Belinda: Challenging (one of the response options) is the biggest.
>
> Sebastian: So write up "challenging is the biggest" so do it in a pie chart and write 12 percent." …
>
> Peter: Yeah, we just take the stuff from here and put it there (on the circle graph).
>
> Sebastian: So do a big thing there and write 12 percent.

What was especially interesting in the children's apparent links between conceptual and metarepresentational competence was their application of percent and percentages, and their efforts in trying to represent 0% and 1%. Of the nine grade 2 student focus groups that progressed to the third year, seven tried to apply their awareness of these ideas in creating their representations. The children had not formally studied this topic but were transferring their learning from their experience with the grade 7 class in the second year (e.g., "Do we have to do percentages like the Year 7s did last year?") and sharing their ideas with those group members who were not involved in the grade 2/7 experience. However, one group who explained that "12% means 12 people" subsequently noted that, "We don't know how to use it."

For the five groups who received zero preferences for one of their response options, considerable debate was held on how to display 0% and 1% For example, one group was experimenting with how to represent their data on their circle graph for the response option to the question, "Why do you like the playground?" The response option, "fits more people than the oval" received zero votes, while the option, "good views" received one preference. After instructing Kim to "Write 12 percent" in one sector of the circle graph (representing 12 responses), Belinda said, "Maybe you could rule a little bit off it…that could be zero percent." She further noted that the response option of good views "only has one percent…" and "has to be really small, like that small." This group further struggled with their display of 0%, claiming that there was insufficient space to label the option of "fits more people than the oval." When their teacher asked, "How can you show 0%?," Belinda responded that, "You should just rub that out…cause that got nothing." But then Kim was puzzled by "How would you do zero?" to which Belinda replied "Rub it out, rub it out."

Another group, however, demonstrated a more advanced understanding of percent, albeit they did not calculate all the percentages correctly. When two group members recommended recording the number of focus question responses (to the question, "Why do you like the Spider Web") in the circle graph segments they had drawn, Hugh disagreed, saying "Do percent". When the research assistant queried the group on how they intended determining this, they explained that they knew that the circle graph represents 100% and that of the 20 votes for their focus question, there were two responses that each received five votes, one that received four, and one that scored six. Hugh explained that instead of recording the actual number of focus question responses, percentages should be shown: "That is 100%, so we needed to do 25%; that's 25 (%), so that should be 24 (%), and that should be 25 (%) and that one should be 26 (%).

Discussion and Concluding Points

The children demonstrated competence in structuring and representing the data they had collected and were readily able to create more than one representation; this was in spite of minimal instruction. They also demonstrated an understanding that their different representations were representing the same data.

The children displayed skills in their use of various inscriptions, labelling vertical and horizontal axes of their bar graphs and the sectors in their circle graphs. Some children's use of space on their bar graphs, as well as all children's use of colour on all their representations, suggests they were aware of the importance of making their graphs readily interpretable. Children were also inventive in how they overcame any obstacles in their representational material, such as using two columns to display data for one response option. The innovative representations, such as the "heart rate monitor" graph, also show young children's creative links with their world and their ability to reason analogically (English, 2004).

The children's display of conceptual and metarepresentational competence was noteworthy, especially in their efforts in creating their circle graphs, where they invented a number of ways to determine the placement of their sectors. Their keenness to use the notion of percent and also their debates on how to represent “0%”were surprising findings. Experiencing the models produced by the grade 7 students 12 months previously generated an interest in this notion for the children. Although the majority of children did not actually calculate correct percentages, they nevertheless were aware of the notion of percent and displayed a basic understanding of how it can be used. The sharing of products was a rich learning experience for both grade levels, providing opportunities for appreciating different approaches to dealing with data and for questioning, explaining, and interpreting the data models of others. Consideration should be given to creating such sharing opportunities across grade levels, with research exploring effective activities and the combined learning that takes place.

Further research on the development of children’s metarepresentational competence is also clearly needed, with consideration given to how we can advance their existing “native”competence (diSessa, 2004) here in connection with their conceptual development.

Acknowledgement

This project reported is supported by a three-year Australian Research Council (ARC) Discovery Grant DP0984178 (2009-2011). Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the author and do not necessarily reflect the views of the ARC. I wish to acknowledge the excellent support provided by the senior research assistant, Jo Macri.

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